

# DEPARTMENT OF MATHEMATICS

BUNDELKHAND COLLEGE JHANSI-284001



COMPLEX ANALYSIS (MCQ) FOR B.A. AND B.SC.  
STUDENTS

BY

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- (1) Let  $(X, d)$  be a metric space. If there exist a number  $K$  such that  $d(x, y) \leq K, \forall x, y \in X$ , then metric space is called:
- (a) Bounded Metric space ★
  - (b) Discrete metric space
  - (c) Usual metric space
  - (d) None of above
- (2) Let  $(X, d)$  be a metric space such that  $d(x, y) = 1$  if  $x \neq y$  and  $d(x, y) = 0$  if  $x = y$  then:
- (a)  $d(x, y) \leq d(x, z) + d(z, y)$  ★
  - (b)  $d(x, y) \geq d(x, z) + d(z, y)$
  - (c)  $d(x, y) > 0$
  - (d) None of above
- (3) If  $d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$  where  $x = (x_1, x_2), y = (y_1, y_2) \in R^2$  then:
- (a)  $d(x, y) \neq d(y, x)$
  - (b)  $(R^2, d)$  is metric space ★
  - (c)  $(R^2, d)$  is not metric space
  - (d) None of above
- (4) For usual metric  $(R, d)$  on  $R$  every singleton set in  $R$  is:
- (a) Open
  - (b) Closed ★
  - (c) Open and Closed
  - (d) None of above

- (5) The unit open ball  $(\mathbb{R}^2, d)$  at the origin is:
- (a)  $\{(x, y) \in \mathbb{R}^2 : |x| + |y| < 1\}$
  - (b)  $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$
  - (c)  $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$  ★
  - (d)  $\{(x, y) \in \mathbb{R}^2 : |x| + |y| > 1\}$
- (6) If  $A$  and  $B$  are subsets of metric space  $(X, d)$  and  $\bar{A}$  denotes the closure of  $A$ , which of the following is false:
- (a)  $A \subset \bar{A}$
  - (b)  $A \subset B \Rightarrow \bar{A} \subset \bar{B}$
  - (c)  $\overline{A \cup B} \subseteq \bar{A} \cup \bar{B}$  ★
  - (d)  $\overline{A \cap B} \subseteq \bar{A} \cap \bar{B}$
- (7) If  $A$  and  $B$  are subsets of metric space  $(X, d)$  and  $A^0$  denotes the interior of  $A$ , which of the following is true:
- (a)  $A^0 \cup B^0 = (A \cup B)^0$
  - (b)  $(A \cup B)^0 \subset A^0 \cup B^0$
  - (c)  $(A \cup B)^0 \supset A^0 \cup B^0$  ★
  - (d) None of these
- (8) Every subset of a discrete metric space  $(X, d)$  is:
- (a) Open
  - (b) Closed
  - (c) Open as well as closed ★
  - (d) None of above

(9) Union of two compact sets in a metric space  $(X, d)$  is:

- (a) Open set
- (b) Closed set
- (c) Compact set ★
- (d) None of above

(10) A compact subset of a metric space  $(X, d)$  is:

- (a) Open set
- (b) Closed set ★
- (c) Unbounded set
- (d) None of above

(11) A set  $R$  of real numbers with usual metric space is:

- (a) Disconnected space
- (b) Connected space ★
- (c) Totally disconnected space
- (d) None of above

(12) Let  $A$  and  $B$  be any two connected sets in a metric space  $X$ , then:

- (a)  $A \cup B$  is always connected
- (b)  $A \cup B$  is not connected
- (c)  $A \cup B$  is connected, if  $A \cap B \neq \phi$  ★
- (d)  $A \cup B$  is connected if  $A \cap B = \phi$

(13) A real numbers set  $E$  is compact if:

- (a)  $E$  is bounded and open
- (b)  $E$  is unbounded and closed

(c)  $E$  is bounded and closed ★

(d)  $E$  is unbounded and open

(14) For a subset  $A$  of a metric space  $(X, d)$ , we have  $\bar{A} =$ :

(a)  $A$  ★

(b)  $\bar{A}$

(c)  $A^c$

(d)  $A^0$

(15) Subset  $A$  of a metric space  $(X, d)$ , we have is open if and only if:

(a)  $A = \bar{A}$  ★

(b)  $A = A^0$

(c)  $A \neq A^0$

(d) None of above

(16) If  $f(z)$  is a analytic function and  $z = re^{i\theta}$ , The Cauchy Riemann

equations are

(a)  $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = \frac{1}{r} \frac{\partial u}{\partial \theta}$

(b)  $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$  ★

(c)  $\frac{\partial u}{\partial r} = -\frac{1}{r} \frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = \frac{1}{r} \frac{\partial u}{\partial \theta}$

(d)  $\frac{\partial u}{\partial r} = -\frac{1}{r} \frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$

(17) Cartesian form of the Cauchy- Riemann equation is:

(a)  $u_x = v_y, u_y = -v_x$  ★

(b)  $v_x = u_y, u_y = -v_x$

(c)  $u_x = v_y, u_x = -v_y$

(d)  $u_y = v_x, u_y = -v_x$

- (18) Which of the following is true?
- (a) Differentiability does not implies continuity
  - (b) Differentiability implies continuity ★
  - (c) Continuity implies differentiability
  - (d) There is no relation between continuity and differentiability
- (19) The function  $f(z) = |z|^2$  has
- (a) One singular point
  - (b) Two singular points
  - (c) Three singular points
  - (d) No singular point ★
- (20) What is the value of  $m$  for which  $f(x, y) = 2x - x^2 + my^2$  is harmonic ?
- (a) 1
  - (b) -1
  - (c) 2 ★
  - (d) -2
- (21) Which of the following function  $f(z)$ , of the complex variable  $z$ , is not analytic at all the points of the complex plane?
- (a)  $f(z) = z^2$
  - (b)  $f(z) = e^z$
  - (c)  $f(z) = \log(z)$  ★
  - (d)  $f(z) = \sin z$

- (22) If  $f(z)$  is an analytic function whose real part is constant then  $f(z)$  is:
- (a) function of  $z$
  - (b) function of  $x$  only
  - (c) function of  $y$  only
  - (d) constant ★
- (23) A function which is analytic everywhere in a complex plane is known as
- (a) Harmonic function
  - (b) differentiable function
  - (c) entire function ★
  - (d) regular function
- (24) The function  $f(z) = xy + iy$  is
- (a) Nowhere analytic
  - (b) Analytic every where
  - (c) Analytic only at origin ★
  - (d) Analytic except at the origin
- (25) The harmonic conjugate of  $u = \frac{y}{x^2+y^2}$
- (a)  $2xy + y + c$
  - (b)  $2xy + 2y + c$
  - (c)  $xy + y + c$
  - (d)  $2xy - y + c$
- (26) Period of  $e^z$  is
- (a)  $-2\pi$

- (b)  $2\pi$  ★
- (c)  $2\pi i$
- (d)  $-2\pi i$
- (27) The value of the integral  $\int_c \frac{dz}{z^2}$  where  $c$  is the positively oriented circle  $z = 2e^{i\theta}$ ,  $(-\pi \leq \theta \leq \pi)$  about the origin is
- (a) 1
- (b) -1
- (c) 0 ★
- (d) 2
- (28) The integral of  $\int_c \frac{dz}{z^2+9}$ , where  $c$  is the unit circle is
- (a) 0 ★
- (b) 1
- (c) 3
- (d) -3
- (29) The integral of  $\int_c \frac{z^2}{z-2} dz$ , where  $c$  is the circle  $|z| = 3$  is
- (a)  $2\pi i$
- (b)  $4\pi i$  ★
- (c)  $8\pi i$
- (d)  $-\pi i$
- (30) The integral of  $\int_c \frac{z^2}{z-i} dz$ , where  $c$  is the circle  $|z| = 2$  is
- (a)  $2\pi i$  ★
- (b)  $4\pi i$

- (c)  $8\pi i$
- (d)  $-\pi i$
- (31) The integral of  $\int_c \frac{1}{3z^2+1} dz$ , where  $c$  is the circle  $|z|=1$  is
- (a)  $\pi i$  ★
- (b)  $-\pi i$
- (c) 0
- (d) 1
- (32) If  $p(z)$  is a polynomial of degree  $n \geq 1$  then it has
- (a)  $n$  zeros ★.
- (b)  $n+1$  zeros
- (c)  $n-1$  zeros
- (d)  $n+2$  zeros
- (33) If a function  $f(z)$  is analytic throughout a simple connected domain  $D$ , then  $\int_c f(z) dz =$
- (a) 0 ★
- (b)  $2\pi i$
- (c)  $2\pi i f(z)$
- (d) 1
- (34) The integral of the function  $\int_c \frac{\cos z}{z} dz$  where  $c$  is the unit circle is
- (a)  $\pi$
- (b)  $\pi i$
- (c)  $-\pi i$
- (d)  $2\pi i$  ★

(35) The integral of the function  $\int_c \frac{\sin z}{z} dz$  where  $c$  is the unit circle is

- (a)  $\pi$
- (b)  $\pi i$
- (c)  $-\pi i$
- (d)  $0$  ★

(36) The integral of the function  $\int_c \frac{e^z}{z} dz$  where  $c$  is the unit circle is

- (a)  $\pi$
- (b)  $\pi i$
- (c)  $-\pi i$
- (d)  $2\pi i$  ★

(37) The integral of the function  $\int_c \frac{z+2}{z-2} dz$  where  $c$  is the unit circle is

- (a)  $\pi$
- (b)  $\pi i$
- (c)  $-\pi i$
- (d)  $0$  ★

(38) The integral of the function  $\int_c \frac{\cos z}{z^2} dz$  where  $c$  is the unit circle is

- (a)  $\pi$
- (b)  $\pi i$
- (c)  $-\pi i$
- (d)  $0$  ★

(39) The integral of the function  $\int_c \frac{\cos z}{2z} dz$  where  $c$  is the unit circle is

- (a)  $\pi$
- (b)  $4\pi i$  ★

- (c)  $-\pi i$
- (d) 0
- (40) The integral of the function  $\int_c e^z \cos z dz$  where  $c$  is the unit circle is
- (a)  $\pi(3 + 2i)$
- (b)  $\frac{\pi}{2}(3 + 2i)$
- (c)  $\frac{\pi}{3}(3 + 2i)$
- (d)  $\frac{\pi}{2}(2 + 3i)$
- (41) If  $f(z)$  is analytic within and on a simple closed positively oriented contour  $c$  and if  $z_0$  is a point interior to  $c$ , then  $\int_c \frac{f(z)}{(z-z_0)^{n+1}}$  equals
- (a)  $\frac{2\pi i}{n!} f(z_0)$
- (b)  $\frac{2\pi i}{n!} f'(z_0)$
- (c)  $\frac{2\pi i}{n!} f''(z_0)$
- (d)  $\frac{2\pi i}{n!} f^{(n)}(z_0)$  ★
- (42) If  $f(z)$  is continuous in a domain  $D$  and if  $\int_c f(z) dz = 0$  for every simple closed positively oriented contour  $c$  in  $D$ , then
- (a)  $f(z)$  is analytic in  $D$  ★
- (b)  $f(z)$  is single valued in  $D$
- (c)  $f(z)$  is constant in  $D$
- (d) None of these
- (43) The converse of Cauchy- integral theorem is if  $f(z)$  is continuous in  $D$
- (a) Euler's theorem
- (b) Morera's theorem ★

- (c) Liouville's theorem
- (d) Goursat's theorem
- (44) Piecewise smooth curve is also known as
- (a) contour ★
- (b) smooth curve
- (c) circle
- (d) regular curve
- (45) Taylor series representation for  $\frac{1}{z}$  about  $z = 1$  is
- (a)  $1 - (1 - z) + (1 - z)^2 - \dots$  ★
- (b)  $1 + (1 - z) + (1 - z)^2 + \dots$
- (c)  $1 - (1 - z) - (1 - z)^2 - \dots$
- (d)  $1 - (1 - z) - (1 - z)^2 - \dots$
- (46) A Maclaurin series is a Taylor series with center
- (a) 0 ★
- (b) 1
- (c) -1
- (d) 2
- (47) Maclaurin series of  $\sin z$  is
- (a)  $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{z^{2n+1}}{(2n+1)!}$
- (b)  $\sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$  ★
- (c)  $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{z^{2n}}{(2n)!}$
- (d)  $\sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$

- (48) The radius of convergence of the series  $\sum_{n=0}^{\infty} \frac{n!}{n^n} z^n$  is
- (a)  $\frac{1}{e}$  ★
  - (b) 0
  - (c)  $e$
  - (d) 1
- (49) The center of the power series  $\sum_{n=0}^{\infty} (z - 4i)^n$  is
- (a)  $4i$
  - (b)  $-4i$  ★
  - (c)  $2i$
  - (d) None of these
- (50) A power series always  $\sum_{n=0}^{\infty} a_n(z - z_0)^n$  converges for
- (a) at all  $z$  which are either real or purely imaginary
  - (b) at all  $z$  with  $|z - z_0| < R$  for some  $R > 0$  ★
  - (c) at least one point  $z$
  - (d) all complex numbers  $z$
- (51) If the principal part of  $f(z)$  at  $z_0$  is zero, then the point  $z_0$  is known as
- (a) removable singular point ★
  - (b) Pole
  - (c) Simple pole
  - (d) None of these
- (52) If the principal part of  $f(z)$  at  $z_0$  has finit terms, then the point  $z_0$  is known as
- (a) removable singular point
  - (b) Pole

- (c) Simple pole
  - (d) essential singular point ★
- (53) The zero of the function  $\frac{z}{\cos z}$  is
- (a) 1
  - (b) 0 ★
  - (c)  $-1$
  - (d)  $\pi$
- (54) The singularity of the function  $\frac{e^z-1}{z}$  is
- (a)  $\pi$
  - (b)  $-\pi$
  - (c) 0 ★
  - (d) 1
- (55) if  $f(z)$  has a pole of order  $m$  at  $z_0$  then  $g(z) = \frac{f(z)}{f'(z)}$  at  $z_0$  has
- (a) a simple pole ★
  - (b) a pole of order  $m$
  - (c) a pole of order  $m + 1$
  - (d) None of these
- (56) The singular point of the function  $\frac{1}{4z-z^2}$  are
- (a) 0,4 ★
  - (b) 0,-4
  - (c) 0,0
  - (d) 0,2

- (57) The nature of the singularity of function  $\frac{1}{\cos z - \sin z}$  at  $z = \frac{\pi}{4}$  is
- (a) removable singularity
  - (b) isolated singularity ★
  - (c) simple pole
  - (d) essential singularity
- (58) Which of the following is related to Cauchy residue theorem?
- (a)  $\int_c f(z) dz = 0$
  - (b)  $\int_c \frac{f(z)}{z-z_0} dz = 2\pi i f(z_0)$
  - (c)  $\int_c f(z) dz = 2\pi i$ (sum of residues) ★
  - (d) None of these
- (59) Analytic function  $f(z) = u + iv$  of which the real part  $u = e^x(x \cos y - y \sin y)$  is
- (a)  $z + e^z$
  - (b)  $ze^z + \sin z$
  - (c)  $ze^z + c$  ★
  - (d) None of above
- (60) If  $f(z)$  is an analytic within and on a closed contour  $C$   $a$  is any point lying in it, then :
- (a)  $2\pi i f(a) = \int \frac{f(z)}{(z-a)^2}$
  - (b)  $2\pi i f'(a) = \int \frac{f(z)}{(z-a)^2}$  ★
  - (c)  $f'(a) = \int \frac{f(z)}{(z-a)^2}$
  - (d) None of these

(61)  $f(z) = \frac{z+2}{(z-1)(z-2)(z-3)}$  has singularities at :

- (a)  $z = 1$
- (b)  $z = 1, 2$
- (c)  $z = 1, 2, 3$
- (d) All true ★

(62) The value of  $\int_C \frac{dz}{(z-1)(z+1)}$  Where  $C$  is  $|z| = 3$ :

- (a) 1
- (b) 0 ★
- (c) -1
- (d) None

(63)  $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} d\theta$  is :

- (a)  $\frac{\pi}{2}$
- (b)  $\pi$
- (c)  $\frac{\pi}{6}$  ★
- (d)  $\frac{\pi}{3}$

(64)  $\int_0^\infty \frac{\sin mx}{x} dx$  is :

- (a)  $\frac{\pi}{2}$  ★
- (b)  $\pi$
- (c)  $\frac{\pi}{6}$
- (d)  $\frac{\pi}{3}$

(65) Residue of  $\frac{z^2}{z^2+1}$  at  $z = i$  is:

- (a)  $\frac{i}{4}$
- (b)  $-\frac{i}{4}$

- (c)  $\frac{i}{2}$  ★
- (d)  $\frac{1}{4}$
- (66) Number of isolated singularities of the function  $f(z) = \frac{1}{\sin \frac{\pi}{z}}$ :
- (a) 1
- (b) 2
- (c) 5
- (d)  $\infty$  ★
- (67) Number of poles of the function  $f(z) = \tan \frac{1}{z}$  is:
- (a) 1
- (b) 2
- (c) 4
- (d)  $\infty$  ★
- (68) Under the transformation  $w = z + 1 - i$ , the image of line  $x = 0$  in w-plane is
- (a)  $u = 1$  ★
- (b)  $u = 0$
- (c)  $v = 1$
- (d)  $v = 0$
- (69) A transformation of type  $w = \alpha z + \beta$  where  $\alpha$  and  $\beta$  are complex constant:
- (a) Translation
- (b) Magnification

(c) Translation and Magnification ★

(d) None

(70) For two complex numbers  $z_1, z_2$  then  $|z_1 - z_2|$  is:

(a)  $\geq |z_1| - |z_2|$  ★

(b)  $\geq |z_1| + |z_2|$

(c)  $= |z_1| - |z_2|$

(d) None of these

(71) A harmonic conjugate of  $u = e^x \sin y$  is:

(a)  $e^y \cos x$

(b)  $-e^x \cos y$  ★

(c)  $-e^{x \cos y + 1}$

(d)  $\frac{e^y}{\sin x}$

(72) If  $|f(z)| \leq M$  on curve  $C$   $|z - z_0| < \rho$  then :

(a)  $|f^{(n)}(z_0)| \leq n! \frac{M}{\rho^n}$  ★

(b)  $|f^{(n)}(z_0)| < n! \frac{M}{\rho^n}$

(c)  $|f^{(n)}(z_0)| \geq n! \frac{M}{\rho^n}$

(d) All of these

(73)  $z = 0$  for the function  $f(z) = \log z$  is :

(a) Isolated singularity ★

(b) Pole

(c) Non-isolated singularity

(d) All of these

- (74) If  $f(z)$  has zero of order  $m$ , then :
- (a)  $f^{(m)} = 0$
  - (b)  $f^{(m-2)} = 0$
  - (c)  $f^{(m-1)} = 0$  ★
  - (d) All of these
- (75) The integral of  $\int_c \frac{z^2}{z-i} dz$ , where  $c$  is the circle  $|z| = 2$  is
- (a)  $-2\pi i$  ★
  - (b)  $4\pi i$
  - (c)  $8\pi i$
  - (d)  $-\pi i$
- (76) The integral of  $\int_c \frac{1}{3z^2+1} dz$ , where  $c$  is the circle  $|z| = 1$  is
- (a)  $\pi i$
  - (b)  $-\pi i$
  - (c)  $0$  ★
  - (d)  $1$
- (77) If  $p(z)$  is a polynomial of degree  $n \geq 1$  then it has
- (a)  $n$  zeros ★
  - (b)  $n + 1$  zeros
  - (c)  $n - 1$  zero
  - (d)  $n + 2$  zeros
- (78) If a function  $f(z)$  is analytic throughout a simple connected domain  $D$ , then  $\int_c f(z) dz =$
- (a)  $0$  ★

(b)  $2\pi i$

(c)  $2\pi i f(z)$

(d) 1

(79) The integral of the function  $\int_c \frac{\cos z}{z} dz$  where  $c$  is the unit circle is

(a)  $\pi$

(b)  $\pi i$

(c)  $-\pi i$

(d)  $2\pi i$  ★

(80) The integral of the function  $\int_c \frac{\sin z}{z} dz$  where  $c$  is the unit circle is

(a)  $\pi$

(b)  $\pi i$

(c)  $-\pi i$

(d) 0 ★

(81) If  $T_1(z) = \frac{z+2}{z+3}$ ,  $T_2(z) = \frac{z}{z+1}$ , then the value of  $T_2^{-1}T_1(z)$  is

(a)  $z-2$

(b)  $z+2$  ★

(c)  $2z$

(d)  $\frac{z+2}{z-3}$

(82) Which is not Magnification

(a)  $w = 10z$

(b)  $w = \frac{1}{8z}$

(c)  $w = 3z$

(d)  $w = z+3$  ★

(83) Under the translation transformation, if  $w = z + (1 - 2i)$ , then  $u$  and

$v$  are

(a)  $u = x; v = y$

(b)  $u = x - 1; v = y - 2$

(c)  $u = x + 1; v = y - 2$  ★

(d)  $u = x + y; v = 0$

(84) The function  $f(z) = \frac{1}{z(z-3)}$  is not analytic at  $z = \dots$

(a) 0, 0

(b) 3, 3

(c) 0, 3 ★

(d) 3,  $\infty$

(85) The real component of the function  $\sin h(e^z)$  is :

(a)  $\cosh(e^x) + \sinh(e^x)$

(b)  $\cosh(e^x) - \sinh(e^x)$

(c)  $\cos(e^x) + \sin(e^x)$

(d)  $\sinh(e^x \cos y) \cos(e^x \sin y)$  ★

(86) If  $w = f(z) = u + iv$  and  $u - v = e^x(\cos y - \sin y)$ , find  $w$  in terms of  $z$ :

(a)  $f(z) = w = e^z + c$  ★

(b)  $f(z) = w = e^{iz} + c$

(c)  $f(z) = w = e^x$

(d)  $f(z) = w = e^y$

(87) Evaluate  $\frac{1}{4z - z^2}$ , when  $0 < |z| < 4$ :

(a)  $\sum_{n=0}^{\infty} z^{n-1}$

$$(b) \sum_{n=0}^{\infty} z^{n-1} 4^{n+1}$$

$$(c) \sum_{n=0}^{\infty} z^{n+1} 4^{n+1}$$

$$(d) \sum_{n=0}^{\infty} \frac{z^{n-1}}{4^{n+1}} \star$$

(88) Expand  $e^z$  in a Taylor's series with region of convergence:

$$(a) \sum_{n=0}^{\infty} \frac{z^n}{n!} \text{ when } |z| < \infty \star$$

$$(b) \sum_{n=0}^{\infty} z^n n! \text{ when } |z| < \infty$$

$$(c) \sum_{n=0}^{\infty} z^{-n} n! \text{ when } |z| > \infty$$

$$(d) \sum_{n=0}^{\infty} z^{-n} \text{ when } |z| = 0$$

(89) If  $c$  is a closed curve with  $z = a$  inside  $c$ , then  $\int_c \frac{dz}{z-a} = \dots :$

$$(a) \pi i$$

$$(b) 2\pi i \star$$

$$(c) 3\pi i$$

$$(d) 4\pi i$$

(90) Evaluate  $\int_0^{1+i} z^2 dz = \dots :$

$$(a) (1+i)^3$$

$$(b) (1-i)^3$$

$$(c) \frac{1}{3}(1+i)^3 \star$$

$$(d) \frac{1}{3}(1-i)^3$$

(91) Evaluate  $|z_1 + z_2|^2 + |z_1 - z_2|^2 = \dots :$

$$(a) 2|z_1|^2 - 2|z_2|^2$$

$$(b) 2|z_1|^2 + 2|z_2|^2 \star$$

$$(c) |z_1|^2 - |z_2|^2$$

$$(d) |z_1|^2 + |z_2|^2$$

- (92) Evaluate  $|z_1 + z_2|^2 + |z_1 - z_2|^2 = \dots$  :
- (a)  $2|z_1|^2 - 2|z_2|^2$
  - (b)  $2|z_1|^2 + 2|z_2|^2$  ★
  - (c)  $|z_1|^2 - |z_2|^2$
  - (d)  $|z_1|^2 + |z_2|^2$
- (93) Modulus and Argument of  $\left(\frac{2+i}{3-i}\right)^2$  :
- (a)  $1/2$  and  $\pi/2$  ★
  - (b)  $1/4$  and  $\pi/4$
  - (c)  $1$  and  $\pi$
  - (d)  $1/4$  and  $\pi/2$
- (94) If  $f(z) = e^x(\cos y + i \sin y)$ , then  $f(z) = \dots$ :
- (a)  $e^{-z}$
  - (b)  $e^z$  ★
  - (c)  $ze^z$
  - (d)  $-ze^{-z}$
- (95) Let  $f(z) = e^{-z^{-4}}$  ( $z \neq 0$ ) and  $f(0) = 0$  is not analytic at  $z = \dots$ :
- (a)  $0$  ★
  - (b)  $1$
  - (c)  $-1$
  - (d)  $\infty$
- (96) Cauchy-Reimann equations are:
- (a)  $u_x = v_y, u_y = -v_x$  ★
  - (b)  $u_x = v_x, u_y = -v_y$

(c)  $u_x = v_y, u_y = v_x$

(d)  $u_x = -v_y, u_y = v_x$

(97) Cauchy-Reimann equations are:

(a)  $v_\theta = ru_r, u_\theta = -rv_r$  ★

(b)  $v_\theta = -ru_r, u_\theta = -rv_r$

(c)  $v_\theta = ru_r, u_\theta = rv_r$

(d)  $v_\theta = u_r, u_\theta = -v_r$

(98) Cauchy-Reimann equations are:

(a)  $v_\theta = ru_r, u_\theta = -rv_r$  ★

(b)  $v_\theta = -ru_r, u_\theta = -rv_r$

(c)  $v_\theta = ru_r, u_\theta = rv_r$

(d)  $v_\theta = u_r, u_\theta = -v_r$

(99) Orthogonal system of two family of curves:

(a)  $u_x v_x + u_y v_y = 0$  ★

(b)  $u_x v_x - u_y v_y = 0$

(c)  $u_x v_y + u_y v_x = 0$

(d)  $u_x v_y - u_y v_x = 0$

(100) Any function of  $x$  and  $y$  possessing continuous partial derivatives of the first and second order is called a harmonic function if it satisfies:

(a) Laplace equation ★

(b) Eulers equation

(c) Lagrangian equation

(d) None of above

(101) Find the analytic function of which the real part  $u = e^{-x} [(x^2 - y^2) \cos y + 2xy \sin y]$ :

- (a)  $(x + iy)^2 e^x (\cos y + i \sin y) + c$
- (b)  $(x + iy)^2 e^x (\cos x + i \sin x) + c$
- (c)  $e^{-x} (\cos x + i \sin x) + c$
- (d)  $(x + iy)^2 e^{-x} (\cos y - i \sin y) + c$  ★

(102) If the function  $u = x^3 - 3xy^2$  is harmonic, then its corresponding analytic function is:

- (a)  $f(z) = z + c$
- (b)  $f(z) = z^3 + c$  ★
- (c)  $f(z) = z^2 + z + c$
- (d)  $f(z) = z^2 - z + c$

(103) Which of the following is not correct for analytic function  $f(z)$  and  $g(z)$  in a region  $R$ :

- (a)  $f(z) + g(z)$  is analytic in  $R$
- (b)  $f(z).g(z)$  is analytic in  $R$
- (c)  $f(z) - g(z)$  is analytic in  $R$
- (d)  $f(z)/g(z)$  is analytic in  $R$  ★

(104) Radius of convergence of the power series  $\sum (\log n)^n z^n$  is:

- (a) 0 ★
- (b) 1
- (c) -1
- (d)  $\infty$

(105) Cauchy's integral formula is:

$$(a) f(z_0) = \frac{1}{2\pi i} \int_c \frac{f(z)}{z - z_0} dz \star$$

$$(b) f(z_0) = \frac{1}{2\pi i} \int_c \frac{f(z)}{z + z_0} dz$$

$$(c) f(z_0) = \frac{1}{2\pi} \int_c \frac{f(z)}{z - z_0} dz$$

$$(d) f(z_0) = \frac{1}{2\pi} \int_c \frac{f(z)}{z + z_0} dz$$

(106) Cauchy's Residue Theorem is:

$$(a) \int_c f(z) dz = 2\pi i R$$

$$(b) \int_c f(z) dz = \sum R$$

$$(c) \int_c f(z) dz = 2\pi i \sum R \star$$

$$(d) \int_c f(z) dz = -2\pi i R$$

(107) Maclaurin's series is:

$$(a) f(z) = \sum_{n=1}^{\infty} \frac{z^n}{n!} f^{(n)}(0)$$

$$(b) f(z) = \sum_{n=1}^{\infty} z^n f^{(n)}(0)$$

$$(c) f(z) = f(0) + \sum_{n=1}^{\infty} \frac{z^n}{n!} f^{(n)}(0) \star$$

$$(d) f(z) = \sum_{n=1}^{\infty} \frac{z^n}{(n-1)!} f^{(n)}(0)$$

$$(108) \int_0^{2\pi} \frac{d\theta}{a + b \cos \theta} =:$$

$$(a) 2\pi \sqrt{a^2 - b^2}, a > b > 0$$

$$(b) 2\pi \sqrt{a^2 + b^2}, a > b > 0$$

$$(c) 2\pi i \sqrt{a^2 - b^2}, a > b > 0$$

$$(d) \frac{2\pi}{\sqrt{a^2 - b^2}}, a > b > 0 \star$$

$$(109) \int_0^{2\pi} \frac{d\theta}{5 + 3 \cos \theta} =:$$

$$(a) 2\pi$$

$$(b) \frac{\pi}{2} \star$$

(c)  $\pi\sqrt{3}$

(d)  $\frac{\pi}{(5+3i)}$

(110)  $\int_0^{2\pi} \frac{d\theta}{a+b\sin\theta} =:$

(a)  $2\pi\sqrt{a^2-b^2}$ ,  $a > b > 0$

(b)  $2\pi\sqrt{a^2+b^2}$ ,  $a > b > 0$

(c)  $2\pi i\sqrt{a^2-b^2}$ ,  $a > b > 0$

(d)  $\frac{2\pi}{\sqrt{a^2-b^2}}$   $a > b > 0$  ★

(111)  $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta} =:$

(a)  $2\pi$

(b)  $\frac{\pi}{2}$  ★

(c)  $\pi\sqrt{3}$

(d)  $\frac{\pi}{(5+3i)}$

(112) If  $w = T_z = \frac{z+2}{z+3}$ , then  $T^{-1}(w)$  is:

(a)  $\frac{2+3w}{w+1}$

(b)  $w(2-3w)$

(c)  $\frac{2-3w}{w-1}$  ★

(d)  $\frac{w}{w-1}$

(113) Polar form of complex number  $-5+5i$  is:

(a)  $5\sqrt{2}e^{\frac{\pi i}{4}}$

(b)  $5\sqrt{2}e^{\frac{-3\pi i}{4}}$

(c)  $5\sqrt{2}e^{\frac{3\pi i}{4}}$  ★

(d) None of above

(114)  $z_1$  and  $z_2$  are two complex numbers then:

(a)  $|z_1 - z_2| = |z_1| - |z_2|$

(b)  $|z_1 - z_2| \leq |z_1| - |z_2|$

(c)  $|z_1 - z_2| \leq |z_1| + |z_2|$  ★

(d)  $|z_1 - z_2| \geq |z_1| + |z_2|$

(115) Derivative of function in polar form of  $w = f(z)$ :

(a)  $\frac{dw}{dz} = \frac{\partial w}{\partial r} e^{i\theta}$  ★

(b)  $\frac{dw}{dz} = -\frac{\partial w}{\partial r} e^{-i\theta}$

(c)  $\frac{dw}{dz} = \frac{\partial w}{\partial \theta} e^{i\theta}$

(d)  $\frac{dw}{dz} = -\frac{\partial w}{\partial \theta} e^{-i\theta}$

(116) Which of the following is correct for the  $w = f(z)$ :

(a)  $\frac{dw}{dz} = \frac{\partial w}{\partial x}$  ★

(b)  $\frac{dw}{dz} = -\frac{\partial w}{\partial x}$

(c)  $\frac{dw}{dz} = \frac{\partial w}{\partial y}$

(d)  $\frac{dw}{dz} = -\frac{\partial w}{\partial y}$

(117) If the power series  $\sum a_n z_n$  is convergent and  $\sum |a_n z_n|$  is not convergent,

the series  $\sum a_n z_n$  is said to be:

(a) divergent

(b) oscillatory

(c) conditionally convergent ★

(d) None of above

(118) Radius of convergent, the series  $\sum n^n z_n$  is:

(a) 0

- (b) 1
- (c)  $\infty$  ★
- (d) None of above

(119) Radius of convergent, the series  $\sum \frac{n}{2^n} z^n$  is:

- (a) 2
- (b) 1
- (c)  $\infty$  ★
- (d) None of above

(120) The value of  $\int_C \frac{1}{z} dz$ , where C is circle  $z = e^{i\theta}$  with  $0 \leq \theta \leq \pi$ :

- (a)  $\pi i$  ★
- (b)  $-\pi i$
- (c)  $2\pi i$
- (d) 0

(121) Which of the following functions does represent the series  $\sum_{n=0}^{\infty} \frac{z^n}{n!}$  for

$$|z| < \infty:$$

- (a)  $\sin z$
- (b)  $\cos z$
- (c)  $e^z$  ★
- (d)  $\log(1+z)$

(122) Number of poles of the function  $f(z) = \tan\left(\frac{1}{z}\right)$ :

- (a) 2
- (b) 4

(c)  $\infty$  ★

(d) *None of above*

(123) Residue of the function  $f(z) = \frac{z+1}{(z-1)(z-2)}$  at  $z = 1$  is:

(a) 2

(b)  $-2$  ★

(c) 1

(d)  $-1$

(124) Residue of the function  $f(z) = \frac{e^z}{z^2(z^2+9)}$  at  $z = 1$  is:

(a) 1

(b)  $1/9$  ★

(c)  $1/2$

(d)  $2/3$

(125) The value of  $\int_0^{\infty} \frac{\log x}{1+x^2}$  is:

(a) 1

(b) 0

(c)  $\pi$

(d)  $\frac{\pi}{2}$  ★

(126) The number of isolated singular points of the function  $f(z) = \frac{z+3}{z^2(z^2+2)}$

is:

(a) 1

(b) 2

(c) 3 ★

(d) 4