

DEPARTMENT OF MATHEMATICS

BUNDELKHAND COLLEGE JHANSI-284001



MEASURE THEORY (MCQ) FOR M.SC. STUDENTS

BY

DR RAM ASREY RAJPUT (ASST.PROF.)

DEPARTMENT OF MATHEMATICS

BUNDELKHAND COLLEGE, JHANSI

UTTAR PRADESH, INDIA-284001

- (1) The union of two outer measurable sets is?
- (a) Not Measurable set.
 - (b) Inner measurable.
 - (c) Outer measurable. ★
 - (d) None of above
- (2) A monotone ring is?
- (a) Not a σ -ring.
 - (b) A σ -ring. ★
 - (c) Both (a) and (b).
 - (d) None of above
- (3) A σ -field is a monotone and monotone field is:
- (a) Not a σ -field.
 - (b) A σ -field. ★
 - (c) Both (a) and (b).
 - (d) None of above
- (4) Let $[a,b]$ be the smallest closed interval containing a closed set F . Then:
- (a) $m(F) = b - a$
 - (b) $m(F) = b - a + m(F')$
 - (c) $m(F) = b - a - m(F')$ ★
 - (d) None of above
- (5) In the sense of Lebesgue if E_1 and E_2 are measurable, then $E_1 \cap E_2$ is:
- (a) Measurable ★
 - (b) Not measurable

- (c) Both (a) and (b)
- (d) None of above
- (6) If E_1 and E_2 are measurable subsets of $[a,b]$, then $m(E_1) + m(E_2)$ is:
- (a) $m(E_1 \cup E_2) - m(E_1 \cap E_2)$
- (b) $m(E_1 \cup E_2) + m(E_1 \cap E_2)$ ★
- (c) $m(E_1 \cup E_2)m(E_1 \cap E_2)$
- (d) None of above
- (7) The intersection of a finite number of measurable sets is:
- (a) Measurable ★
- (b) Not measurable
- (c) Both (a) and (b)
- (d) None of above
- (8) Two functions f and g defined on the same set E are said to be equivalent if:
- (a) $m[E(f = g)] = 0$
- (b) $m[E(f \neq g)] = 0$ ★
- (c) Both (a) and (b)
- (d) None of above
- (9) Lebesgue integral of function $f(x)$ over E is defined as the plane measure of $\Omega(f, E)$:
- (a) $\int_E f(x) = m[\Omega(f, E)]$
- (b) $\int_E f(x)dx = m[\Omega(f, E)]$ ★

- (c) $\int_E f(x)dx = m[\Omega f, E]$
- (d) None of above
- (10) The common value of members U and V is called Lebesgue integral of $f(x)$ over E :
- (a) $\int_E f(x) =$ common value of members U and V ★
- (b) $\int_a f(x) =$ common value of members U and V
- (c) $\int_\infty f(x) =$ common value of members U and V
- (d) None of above
- (11) If a bounded measurable function f satisfies the inequality $\alpha(E) \leq \int_E f(x)dx \leq \beta(E)$:
- (a) First mean value theorem ★
- (b) D. F. Egoroff's theorem
- (c) Frechet theorem
- (d) None of above
- (12) A sequence $\{f_n\}$ of measurable function is converge almost uniformly to a measurable function f defined on a measurable set E , each $\epsilon > 0$, \exists a measurable set $A \subset E$ with $m(A) < \epsilon$, then $\{f_n\}$ coverge to f uniformly on $E - A$:
- (a) First mean value theorem
- (b) Frechnet theorem
- (c) D. F. Egoroff's theorem ★
- (d) None of these

- (13) Let f be a measurable function defined on E . Then for each $\epsilon > 0$, \exists a closed set $F \subset E$ with $m(E - F) < \epsilon$, f is continuous on F :
- (a) First mean value theorem
 - (b) Lusin theorem ★
 - (c) Frechnet theorem
 - (d) None of these
- (14) Let f be a measurable function defined on E . Then \exists a sequence $\{g_n\}$ of continuous of $R : g_n \rightarrow f$ a.e. on E :
- (a) First mean value theorem
 - (b) Lusin theorem
 - (c) Frechnet theorem ★
 - (d) None of these
- (15) Let $\{f_n\}$ be a sequence of Lebesgue integrable functions is said to be converge f :
- (a) $\lim_{n \rightarrow 0} \int_E |f_n + f| = 0$
 - (b) $\lim_{n \rightarrow 0} \int_E |f_n - f| = 0$
 - (c) $\lim_{n \rightarrow \infty} \int_E |f_n - f| = 0$ ★
 - (d) None of these
- (16) Let $\{f_n\}$ be a sequence of measurable defined over a measurable set E . Let f be a measurable functions defined on E : isthen the sequence $\{f_n\}$ is said to be converge f :
- (a) $f(x) < \infty$ a.e. on the set E
 - (b) $\lim_{n \rightarrow \infty} m(|f_n - f| \geq \epsilon) = 0$

(c) Both (a) and (b) ★

(d) None of these

(17) Let $\{f_n\}$ be a sequence of measurable defined over a measurable set E

and if then f is measurable on E :

(a) $f_n(x) = f(x)$ on E

(b) $\lim_{n \rightarrow \infty} (x) = f(x)$ on E

(c) $\lim_{n \rightarrow \infty} (x) = f(x)$ a.e. on E ★

(d) None of these

(18) Let $\{f_n\}$ be a sequence of functions which converges in measure to

the function f on a measurable set E . Then \exists a subsequence which converges to the function of a.e.:

(a) F. Riesz theorem ★

(b) Frechet theorem

(c) D. F. Egoroff's theorem

(d) None of these

(19) Let $\{f_n\}$ be a sequence of measurable defined over a measurable set E

such that $f_n(x) < \psi(x), \forall x \in E$ and $\forall n \in N$, where ψ is integrable over E and the sequence $\langle f_n \rangle$ converges in measure to a measurable function f on E , then:

(a) $\int_E f(x)dx = \int_E f_n(x)dx$

(b) $\int_E f(x)dx = \lim_{n \rightarrow \infty} \int_E f_n(x)dx$ ★

(c) $\int_E f(x)dx = \lim_{n \rightarrow 0} \int_E f_n(x)dx$

(d) None of these

(20) Let $\{f_n\}$ be a non-decreasing sequence of integrable functions defined over a measurable set E . Let $\lim_{n \rightarrow \infty} f_n$ be integrable set E . Then

$$\lim_{n \rightarrow \infty} \int_E f_n(x) dx = \int_E \lim_{n \rightarrow \infty} f_n(x):$$

(a) Fatou's Lemma ★

(b) Frechnet theorem

(c) Beppo-Levies theorem

(d) None of these

(21) If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, then $A \triangle B$ is:

(a) $\{1, 2, 3, 4\}$

(b) $\{3, 4, 5, 6\}$

(c) $\{3, 4\}$

(d) $\{1, 2, 5, 6\}$ ★

(22) In distributive law $X = (A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ and $Y =$

$(A \cap B) \cup C = (A \cup C) \cap (A \cup C)$, then:

(a) X is correct

(b) Y is correct

(c) x and Y is correct ★

(d) None of these

(23) A sequence $\langle a_n \rangle$ is said to be strictly monotonic increasing if:

(a) $a_{n+1} < a_n \forall n \in N$

(b) $a_{n+1} < a_n \forall n \notin N$

(c) $a_{n+1} > a_n \forall n \in N$ ★

(d) $a_{n+1} > a_n \forall n \notin N$

- (24) The limit point of the set $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$ is:
- (a) 1
 - (b) $\frac{1}{2}$
 - (c) 0 ★
 - (d) ∞
- (25) Sum of cardinal numbers $|P| = p, |Q| = q$ and $P \cap Q = \phi$, Then:
- (a) $P + Q = |p + q|$
 - (b) $p + q = |P \cup Q|$ ★
 - (c) Both (a) and (b)
 - (d) None of these
- (26) Complete Lattice R is:
- (a) $R \cup \{-\infty, \infty\}$ ★
 - (b) $R \cap \{-\infty, \infty\}$
 - (c) Both (a) and (b)
 - (d) None of these
- (27) If A is a σ -algebra of subsets of X and m is increase on A , then triple (X, A, m) is called:
- (a) Triple space
 - (b) Measurable space ★
 - (c) Measurable set
 - (d) None of these
- (28) Any set A is said to be measurable w.e.f outer measurable function m_0 if:

(a) $m_0(T) = m_0(A \cap T) + m_0(A' \cap T)$ ★

(b) $m_0(T) = m_0(A \cap T) - m_0(A' \cap T)$

(c) $m_0(T) = m_0(A \cup T) + m_0(A' \cup T)$

(d) $m_0(T) = m_0(A \cup T) - m_0(A' \cup T)$

(29) Set A is outer measurable:

(a) If A' is outer measurable

(b) Iff A' is outer measurable ★

(c) If A' is inner measurable

(d) Iff A' is inner measurable

(30) If A and B are two sets such that $A \subseteq B$, then:

(a) $m^*(A) \geq m^*(B)$

(b) $m^*(A) \leq m^*(B)$ ★

(c) $m^*(A) = m^*(B)$

(d) All of these

(31) Which of the following statements is false for a Lebesgue Outer Measure

of an arbitrary set:

(a) $m^*(A) \geq 0 \forall A$

(b) If $A \subseteq B$, then $m^*(A) \leq m^*(B)$

(c) $m^*(A + x) = m^*(A) \forall x \in \mathbb{R}$

(d) All of these ★

(32) Identify the wrong statement:

(a) Cantor set is uncountable ★

(b) Cantor set is open

- (c) Cantor set is measurable
 - (d) Cantor set is nonempty
- (33) Which is a Measurable set:
- (a) Open set
 - (b) Closed set
 - (c) Borel set
 - (d) All of above ★
- (34) The outer measure of a countable set is:
- (a) 0 ★
 - (b) 1
 - (c) ∞
 - (d) Not exist
- (35) The outer measure of Cantors set is:
- (a) 0 ★
 - (b) 1
 - (c) ∞
 - (d) Not exist
- (36) Which property is called countable subadditivity of outer measure:
- (a) $m^*(\cup_i E_i) \leq \sum_i m^* E_i$ ★
 - (b) $m^*(\cup_i E_i) \geq \sum_i m^* E_i$
 - (c) $m^*(\cup_i E_i) = \sum_i m^* E_i$
 - (d) None of these

(37) If $\{E_k\}_{k=1}^{\infty}$ is an ascending collection of measurable sets and $E =$

$\cup_{k=1}^{\infty} E_k$ then $m(E)$ is equal to:

- (a) 0
- (b) $\sum_{k=1}^{\infty} m(E_k)$
- (c) ∞
- (d) $\lim_{k \rightarrow \infty} m(E_k)$ ★

(38) If $\{E_k\}_{k=1}^{\infty}$ is a descending collection of measurable sets and $E =$

$\cap_{k=1}^{\infty} E_k$ then $m(E)$ is equal to:

- (a) 0
- (b) $\sum_{k=1}^{\infty} m(E_k)$
- (c) ∞
- (d) $\lim_{k \rightarrow \infty} m(E_k)$ ★

(39) Which of the following statements is true for a Lebesgue measure:

- (a) countable additivity
- (b) Monotonicity
- (c) translation invariant
- (d) All the above are true ★

(40) Which of the following statement is true:

- (a) There exists a non-measurable subset of real numbers
- (b) There exists a measurable set which is not a Borel set.
- (c) There exists an uncountable set having measure zero.
- (d) All the above are true ★

- (41) Which of the following statement is true:
- (a) There exists a non-measurable subset of real numbers
 - (b) There exists a measurable set which is not a Borel set.
 - (c) There exists an uncountable set having measure zero.
 - (d) All the above are true ★
- (42) Which of the following is an algebra on $X = \{1, 2, 3\}$:
- (a) $\{\phi\}$
 - (b) $\{\phi, \{1\}, \{1, 2\}\}$
 - (c) $\{\phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ ★
 - (d) All the above are true
- (43) If A and B are two sets such that $m(A) = 0$, then:
- (a) $m(A \cup B) = m(A)$
 - (b) $m(B) = m(A)$
 - (c) $m(A \cup B) = m(B)$ ★
 - (d) $m(A \cup B) \geq m(B)$
- (44) Which of the following statement is true:
- (a) F_σ set is measurable
 - (b) G_δ set is measurable
 - (c) Borel set is measurable
 - (d) All of above ★
- (45) The outer measure of empty set is:
- (a) 0 ★
 - (b) 1

- (c) ∞
 - (d) None of these
- (46) $m(\{k\}) = \dots$ where k is a real number:
- (a) 0 ★
 - (b) 1
 - (c) ∞
 - (d) None of these
- (47) If Ω is an algebra and $A, B \in \Omega$, then:
- (a) $A - B \in \Omega$
 - (b) $A \cup B \in \Omega$
 - (c) $A \cap B \in \Omega$
 - (d) All of these ★
- (48) F_σ set is the:
- (a) intersection of a countable collection of open sets
 - (b) intersection of a countable collection of closed sets
 - (c) union of a countable collection of open sets
 - (d) union of a countable collection of closed sets
- (49) G_δ set is the:
- (a) intersection of a countable collection of open sets ★
 - (b) intersection of a countable collection of closed sets
 - (c) union of a countable collection of open sets
 - (d) union of a countable collection of closed sets

(50) Which one is not equivalent to the measurability of any set of real numbers E :

(a) For each $\epsilon > 0$, there is an open set O containing E for which

$$m(O - E) < \epsilon \star$$

(b) For each $\epsilon > 0$, there is a closed set F containing E for which

$$m(F - E) < \epsilon$$

(c) For each $\epsilon > 0$, there is an open set O containing E for which

$$m(E - F) < \epsilon$$

(d) None of these

(51) For any set E and any $\epsilon > 0$, there exists an open set O containing E for which:

(a) $m(O) \leq m(E) + \epsilon \star$

(b) $m(O) \geq m(E) + \epsilon$

(c) $m(O) \leq m(E) - \epsilon$

(d) None of these

(52) Let f be an extended real valued function defined on a measurable domain E . If f is Lebesgue measurable, then for each real number c , which of the following statements is true:

(a) $\{x \in E : f(x) > c\}$ is measurable

(b) $\{x \in E : f(x) < c\}$ is measurable

(c) $\{x \in E : f(x) = c\}$ is measurable

(d) All of these \star

- (53) Which one of the following is false:
- (a) A monotone function that is defined on an interval is not measurable. ★
 - (b) Let f be an extended measurable real-valued function on E and $f = g$ a.e. on E , then g is measurable on E .
 - (c) A monotone function that is defined on an interval is measurable.
 - (d) A real-valued function that is continuous on its measurable domain is measurable.
- (54) Let f be an extended real valued function defined on a measurable domain E . Then f is measurable if and only if for each open set O , $f^{-1}(O) = \{x \in E : f(x) \in O\}$ is:
- (a) Open ★
 - (b) measurable
 - (c) closed
 - (d) none of these
- (55) Let f and g be measurable functions on E that are finite a.e. on E . Then which of the following statements is false.:
- (a) For $\alpha, \beta \in \mathbf{R}$, $\alpha f + \beta g$ is measurable on E .
 - (b) $(f + g)^2$ is measurable on E
 - (c) fg is measurable on E
 - (d) All of these ★
- (56) The characteristic function χ_A is measurable if and only if:
- (a) A has measure 0

- (b) A has infinite measure
 - (c) A is measurable ★
 - (d) All of these
- (57) Which one of the following is false:
- (a) If f is measurable on E , then $|f|$ is measurable ★
 - (b) If f is measurable on E , then f^+ is measurable.
 - (c) If f is measurable on E , then f^- is measurable.
 - (d) None of these
- (58) Which one of the following is false:
- (a) If f is measurable on E , then $|f|$ is measurable ★
 - (b) If f is measurable on E , then f^+ is measurable.
 - (c) If f is measurable on E , then f^- is measurable.
 - (d) None of these
- (59) Let $\{f_n\}$ be a sequence of nonnegative measurable functions on E . If $\{f_n\} \rightarrow f$ pointwise a.e. on E , then:
- (a) $\int_E f = \liminf \int_E f_n$ ★
 - (b) $\int_E f \geq \liminf \int_E f_n$
 - (c) $\int_E f \leq \liminf \int_E f_n$
 - (d) None of these
- (60) Let f be a nonnegative measurable function on E . Then $\int_E f = 0$ if and only if:
- (a) $f > 0$ a.e. on E ★
 - (b) $f = 0$ a.e. on E

- (c) $f < 0$ a.e. on E
 - (d) None of these
- (61) A real-valued function ϕ defined on a measurable set E is simple if:
- (a) It is measurable
 - (b) It assumes only a finite number of values
 - (c) It is measurable and assumes only a finite number of values ★
 - (d) None of these
- (62) A nonnegative measurable function f on a measurable set E is said to be integrable over E if:
- (a) $\int_E f = 0$
 - (b) $\int_E f > 0$
 - (c) $\int_E f < \infty$ ★
 - (d) None of these
- (63) Let the nonnegative function f be integrable over E. Then:
- (a) f is constant a.e. on E
 - (b) f is 0 a.e. on E
 - (c) f is finite a.e. on E ★
 - (d) None of these
- (64) For an extended real-valued function f on E, the positive part f^+ of f is:
- (a) $f^+(x) = \max\{f(x), 0\}$ ★
 - (b) $f^+(x) = \max\{-f(x), 0\}$

$$(c) f^+(x) = -\max\{f(x), 0\}$$

$$(d) f^+(x) = \min\{f(x), 0\}$$

(65) For an extended real-valued function f on E , the positive part f^+ of f is:

$$(a) f^-(x) = \min\{-f(x), 0\}$$

$$(b) f^-(x) = \max\{-f(x), 0\} \star$$

$$(c) f^-(x) = -\max\{f(x), 0\}$$

$$(d) f^-(x) = \min\{f(x), 0\}$$

(66) If $|f|$ is integrable over E , then the integral of f over E is given by:

$$(a) \int_E f = \int_E f^+ + \int_E f^- \star$$

$$(b) \int_E f = \int_E f^+ - \int_E f^-$$

$$(c) \int_E f = -\int_E f^+ + \int_E f^-$$

(d) None of these

(67) A pair (X, M) consisting of a set X and a σ -algebra M is called a:

(a) Measurable space \star

(b) σ -finite space

(c) Measure Space

(d) Complete Measure Space

(68) Let (X, M, μ) be a measure space. A subset E of X is called measurable

if:

$$(a) \mu(E) = 0$$

$$(b) \mu(E) \neq 0$$

(c) $E \in M$ ★

(d) $E \notin M$

(69) Let (X, M, μ) be a measure space with measure μ is called:

(a) Topological Space

(b) Measure Space

(c) σ -finite space ★

(d) Complete Measure Space

(70) Let (X, M, μ) be a measure space. Then for any finite disjoint collection

$\{E_n\}$ of measurable sets $\mu(\cup_{i=1}^n E_i) = \sum_{i=1}^n \mu(E_i)$ This property is known

as:

(a) Monotonicity

(b) Finite Additivity ★

(c) Excision

(d) Continuity of Measure

(71) Let (X, M, μ) be a measure space. Then for any countable disjoint collection

$\{E_n\}$ of measurable sets $\mu(\cup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} \mu(E_i)$ This property

is known as:

(a) Countable Additivity ★

(b) Monotonicity

(c) Countable Monotonicity

(d) Continuity of Measure

(72) Which of the following sets is Lebesgue measurable

(a) Any open set in \mathbb{R}

- (b) Any closed set in \mathbb{R}
 - (c) Any countable union of open intervals
 - (d) All of the above ★
- (73) The Monotone Convergence Theorem applies to which type of functions
- (a) Increasing sequence of measurable functions ★
 - (b) Decreasing sequence of measurable functions
 - (c) Continuous functions only
 - (d) Differentiable functions only
- (74) Which of the following is true about the null set in measure theory
- (a) It has measure zero
 - (b) It is measurable
 - (c) It can be ignored in integration
 - (d) All of the above ★
- (75) The Fatou's Lemma provides an inequality involving:
- (a) Limit inferior of integrals ★
 - (b) Limit superior of integrals
 - (c) Limit inferior of functions
 - (d) Limit superior of functions
- (76) Which of the following is NOT a property of a sigma-algebra
- (a) Closed under countable unions
 - (b) Closed under complements
 - (c) Closed under finite intersections
 - (d) Closed under arbitrary unions ★

- (77) The Lebesgue Dominated Convergence Theorem requires:
- (a) Pointwise convergence and domination by an integrable function ★
 - (b) Uniform convergence
 - (c) Monotone convergence
 - (d) None of the above
- (78) Which of the following is a measure on a measurable space
- (a) A function that assigns a real number to each subset
 - (b) A function that assigns a non-negative extended real number to each measurable set and is countably additive ★
 - (c) A function that assigns a complex number to each measurable set
 - (d) A function that assigns a probability to each event
- (79) The Lebesgue integral generalizes which of the following integrals
- (a) Riemann integral
 - (b) Stieltjes integral
 - (c) Both (a) and (b) ★
 - (d) None of the above
- (80) Which statement is false
- (a) Difference of two measurable sets is also measurable.
 - (b) Union of two measurable sets is measurable.
 - (c) Both (a) and (b)
 - (d) None of the above ★
- (81) Which statement is false
- (a) Every countable set is measurable.

- (b) Every singleton set is measurable.
 - (c) Both (a) and (b)
 - (d) None of the above ★
- (82) Which statement is true. If $A \Delta B = 0$, then:
- (a) $m(A) \neq m(B)$
 - (b) $m(A) = m(B)$ ★
 - (c) Both (a) and (b)
 - (d) None of the above